

Combining Instance-Based Learning and Logistic Regression for Multilabel Classification



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Introduction

Multilabel Classification: Given an instance space X, a finite set of labels L, and training data $\{(x_i, L_i)\} \subset X \times 2^L$, the goal is to learn a classifier $h: X \to 2^L$.

ECML PKDD 2009 (Journal) Paper Notification [491] Conference | X Research | X | Alek Kolcz to me, eyke Show details Jun 6 Reply | X | Dear Weiwei and Eyke, Conference Party We are pleased to inform you that your paper Programming Classification" Research

- most current multilabel classifiers are model-based
- instance-based classifiers (e.g. MLKNN) showed potential but have not been fully exploited yet (ignorance of correlation between labels)

Instance-Based Learning as Logistic Regression

Key idea: Consider the labels of its neighbors as "extra features" of an instance.

age	weight	height	sex	w.child	# 🌑	# 🔍	# 🖭	9	2	•
26	62	1.83	male	no	1/3	0	1	1	0	1
16	45	1.65	female	no	0	1	1/3	0	1	0
28	85	1.90	male	yes	2/3	0	1	1	0	0
-										
27	50	1.63	male	yes	2/3	1/3	1/3	?	?	?

Binary case

Consider query instance $\mathbf{x_0}$, distance $\delta_i \stackrel{\mathrm{df}}{=} \Delta(\mathbf{x}_0, \mathbf{x}_i)$, posterior probability $\pi_0 \stackrel{\mathrm{df}}{=} \mathbf{P}(y_0 = +1 \,|\, y_i)$:

$$\frac{\pi_0}{1-\pi_0} = \frac{\mathbf{P}(y_t \,|\, y_0 = +1)}{\mathbf{P}(y_t \,|\, y_0 = -1)} \cdot \frac{p_0}{1-p_0} = \rho \cdot \frac{p_0}{1-p_0}$$

$$\log\left(\frac{\pi_0}{1-\pi_0}\right) = \log(\rho) + \underbrace{\log(p_0) - \log(1-p_0)}_{\omega_0}$$

For example, we can define $\
ho =
ho(\delta) \stackrel{\mathrm{df}}{=} \exp\Big(y_i \cdot \frac{lpha}{\delta}\Big).$

Now consider the whole neighborhood of **x₀**:

$$\log\left(\frac{\pi_0}{1-\pi_0}\right) = \omega_0 + \alpha \sum_{\mathbf{x}_i \in \mathcal{N}(\mathbf{x}_0)} \frac{y_i}{\delta_i} = \omega_0 + \alpha \cdot \omega_+(\mathbf{x}_0)$$

Multilabel case

We solve one logistic regression problem for each label.

$$\log\left(\bigotimes_{\mathbf{x}_0}\right) = \omega_0 + \alpha_{\bigodot} \cdot \omega_{+\bigodot}(\mathbf{x}_0) + \alpha_{\bigodot} \cdot \omega_{+\bigodot}(\mathbf{x}_0) + \alpha_{\bigodot} \cdot \omega_{+\bigodot}(\mathbf{x}_0)$$
 bias term To what extent does the presence of label basketball in the neighborhood increase the probability that football is relevant for the query?

Results

dataset	domain	#inst.	#attr.	#labels	card.
emotions	music	593	72	6	1,87
image	vision	2000	135	5	1,24
genbase	biology	662	1186 ⁽ⁿ⁾	27	1,25
mediamill	multimedia	5000	120	101	4,27
reuters	text	7119	243	7	1,24
scene	vision	2407	294	6	1,07
yeast	biology	2417	103	14	4,24

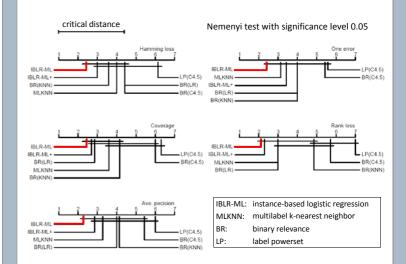
Evaluation metrics

- lacksquare Hamming loss $= rac{1}{|\mathcal{L}|} ig| h(\mathbf{x}) \, \Delta \, L_{\mathbf{x}} ig|$
- one error = $\begin{cases} 1 & \text{if } \arg \max_{\lambda \in \mathcal{L}} f(\mathbf{x}, \lambda) \notin L_{\mathbf{x}} \\ 0 & \text{otherwise} \end{cases}$
- coverage = $\max_{\lambda \in L_{\tau}} \operatorname{rank}_{f}(\mathbf{x}, \lambda) 1$
- rank loss $=\frac{\#\{(\lambda,\lambda')\,|\,f(\mathbf{x},\lambda)\leq f(\mathbf{x},\lambda'),(\lambda,\lambda')\in L_\mathbf{x}\times\overline{L_\mathbf{x}}\}}{|L_\mathbf{x}||\overline{L_\mathbf{x}}|}$
- average precision

$$= \frac{1}{|L_{\mathbf{x}}|} \sum_{\boldsymbol{\lambda} \in L_{\mathbf{x}}} \frac{|\{\lambda' \mid \mathrm{rank}_f(\mathbf{x}, \lambda') \leq \mathrm{rank}_f(\mathbf{x}, \lambda), \lambda' \in L_{\mathbf{x}}\}|}{\mathrm{rank}_f(\mathbf{x}, \lambda)}$$

Two-step statistical test

- 1. Test if all methods perform equally (Friedman test)
- 2. If not, compare learners in a pairwise way (Nemenyi test)



Our Contribution

- A new instance-based multilabel learning method,
- which is based on a formalization of instance-based classification as logistic regression (combination of model-based and instance-based learning),
- takes the correlation between labels into account and represents it in an easily interpretable way.