Top-k Selection based on Adaptive Sampling of Noisy Preferences

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Value-based Top-k Selection (TKS) (Maron&Moore, 1994)

Ground Truth:
- $E(X_1) = \mu_1$
- $E(X_2) = \mu_2$
- $E(X_3) = \mu_3$
- $E(X_4) = \mu_4$
- $E(X_5) = \mu_5$

Top-K Set:
- $o_2$
- $o_5$
- $o_4$
- $o_1$
- $o_3$

Sampling Values:
- $x_2 = 5.2$
- $x_3 = 9.1$
- $x_1 = 8.5$

Estimation:

Hoeffding-Race:

Expected Value:
- $X_1$
- $X_2$
- $X_3$
- $X_4$
- $X_5$
Preference-based Top-k Selection (TKS)

- Pairwise preferences over the set of options
- Four possible outcomes of a single pairwise comparison between $o_i$ and $o_j$:
  - $o_i \prec o_j \Rightarrow Y_{i,j} = 0$ ((strict) preference for $o_j$)
  - $o_i \succ o_j \Rightarrow Y_{i,j} = 1$ ((strict) preference for $o_i$)
  - $o_i \sim o_j \Rightarrow Y_{i,j} = 1/2$ (indifference)
  - $o_i \perp o_j \Rightarrow Y_{i,j} = 1/2$ (incomparability)
- $y_{i,j} = \mathbb{E}[Y_{i,j}]$
  - if $y_{i,j} > 1/2$ then $o_i$ is preferred to $o_j$
  - It can be estimated on the basis of a finite sample

\[ y_{i,j} \approx \bar{y}_{i,j} = \frac{1}{n} \sum_{\ell=1}^{n} y_{i,j}^{\ell} \]
Preference-based Top-k Selection (TKS)

STOCHASTIC PAIRWISE PREFERENCES

GROUND TRUTH

GROUNDED TRUTH

PREFERENCE-BASED RACING

PREFERENCE-BASED RACING

RANKING PROCEDURE

TOP-K SET

TARGET RANKING

SAMPLING PREFERENCES

ESTIMATION

ESTIMATION

\begin{align*}
& o_1 \succ o_3 \quad (Y_{1,3} = 0) \\
& o_1 \sim o_4 \quad (Y_{1,4} = 1/2) \\
& o_4 \succ o_2 \quad (Y_{2,4} = 1) \\
& o_2 \succ o_1 \quad (Y_{1,2} = 1) \\
& o_1 \sim o_3 \quad (Y_{1,3} = 1/2) \\
& \vdots & \vdots
\end{align*}
Ranking procedures

\[ y_{i,j} = E[Y_{i,j}] \]

1. **Copeland’s ranking** (CO): \( o_i \prec^{\text{CO}} o_j \) if and only if \( d_i < d_j \), where

\[ d_i = \#\{ k \in [K] | 1/2 < y_{i,k} \} \]

- An option \( o_i \) is preferred to \( o_j \) whenever \( o_i \) “beats” more options than \( o_j \) does.

2. **Sum of expectations** (SE) ranking: \( o_i \prec^{\text{SE}} o_j \) if and only if

\[
y_i = \frac{1}{K-1} \sum_{k \neq i} y_{i,k} < \frac{1}{K-1} \sum_{k \neq j} y_{j,k} = y_j.
\]

3. The idea of the **Random walk** (RW) ranking is to handle the matrix \( Y = [y_{i,j}]_{K \times K} \) as a transition matrix \( S \) of a Markov chain and order the options based on its stationary distribution.
Thanks!