

# Cooperation, Power and Justice

Cooperative Game Theory Tutorial

by Yoram Bachrach

Presenter: Weiwei Cheng



# Treasure Island

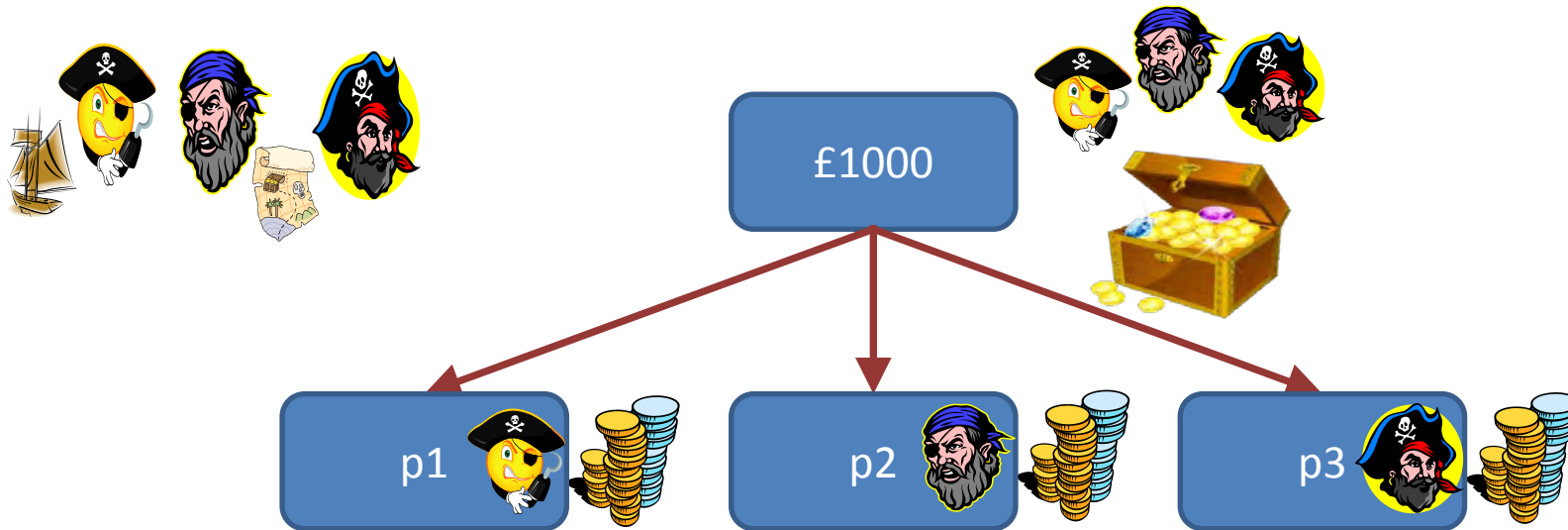


£200



£1000




# Treasure Island – Sharing Rewards



– **Stable** or **Shaky**?



– Is it **Fair**?

- requires  
- very valuable 

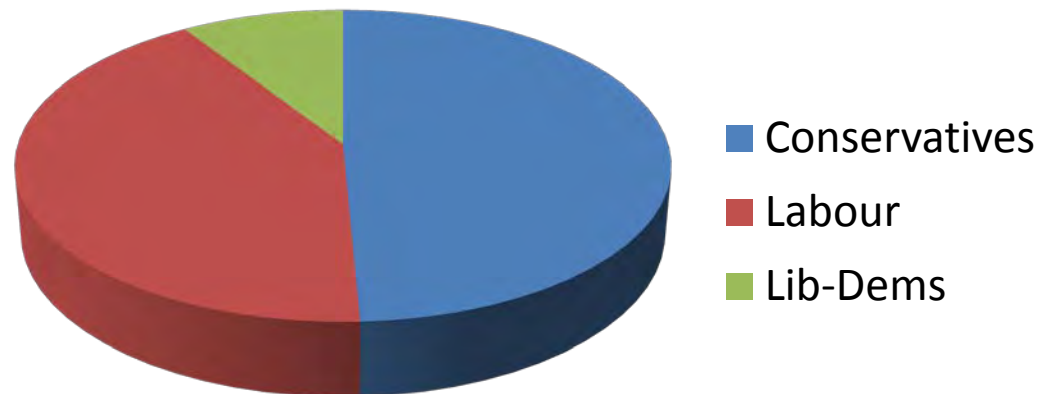


# UK Elections 2010: Budgets and Politics

Conservatives	Labour	Lib-Dems
306	258	57



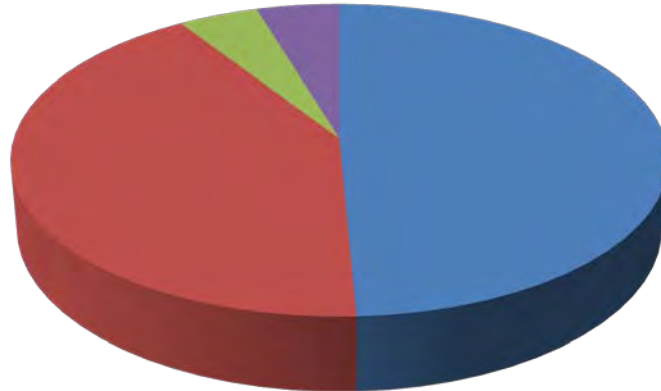
## Seats



# An Alternate Universe

Conservatives	Labour	Liberals	Democrats
306	258	28	29

Seats



- Conservatives
- Labour
- Liberals
- Democrats



# Cooperative Games



Cooperation

Cannot achieve goal alone  
Synergies  
Coordination










Competition

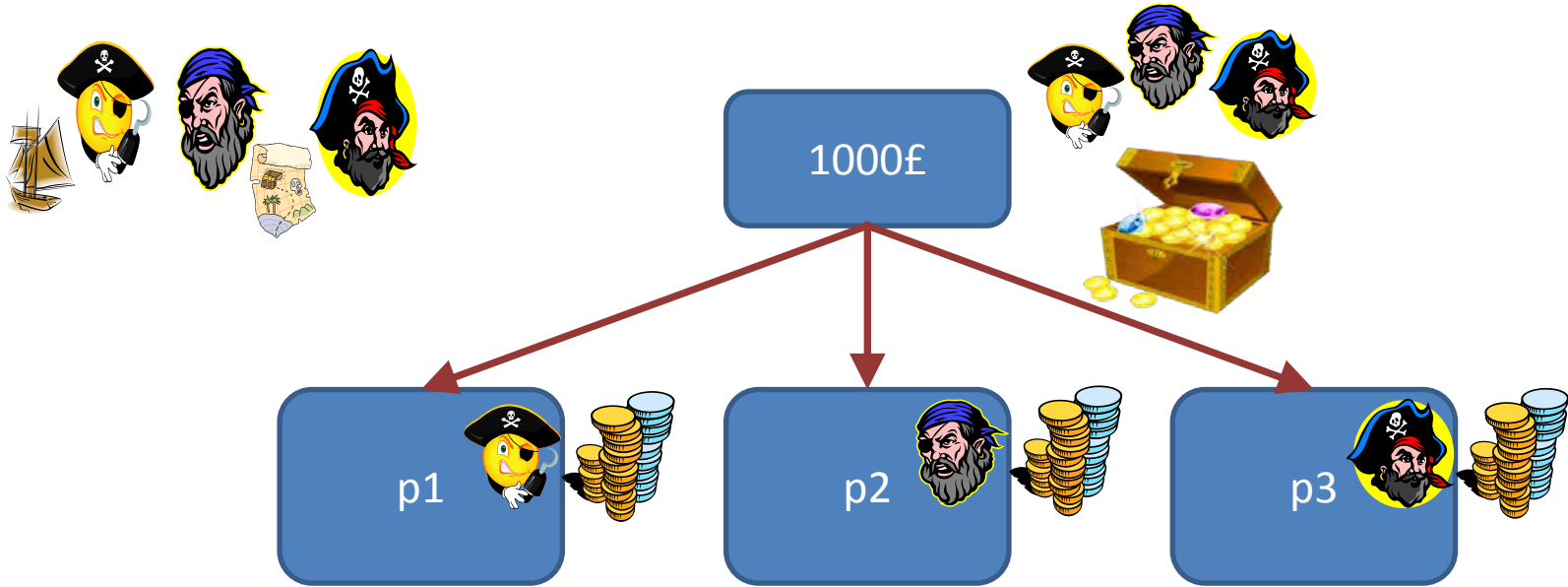
Maximize share of rewards  
Minimize cost  
Increase influence




# Treasure Island

Coalition: C	Value: $v(C)$
$\emptyset$	$v(\emptyset) = 0$
	$v(\{1\}) = 0$
	$v(\{2\}) = 0$
	$v(\{3\}) = 0$
	$v(\{1,2\}) = 0$
	$v(\{1,3\}) = 0$
	$v(\{2,3\}) = 200$
	$v(\{1,2,3\}) = 1000$

# Treasure Island – Imputations

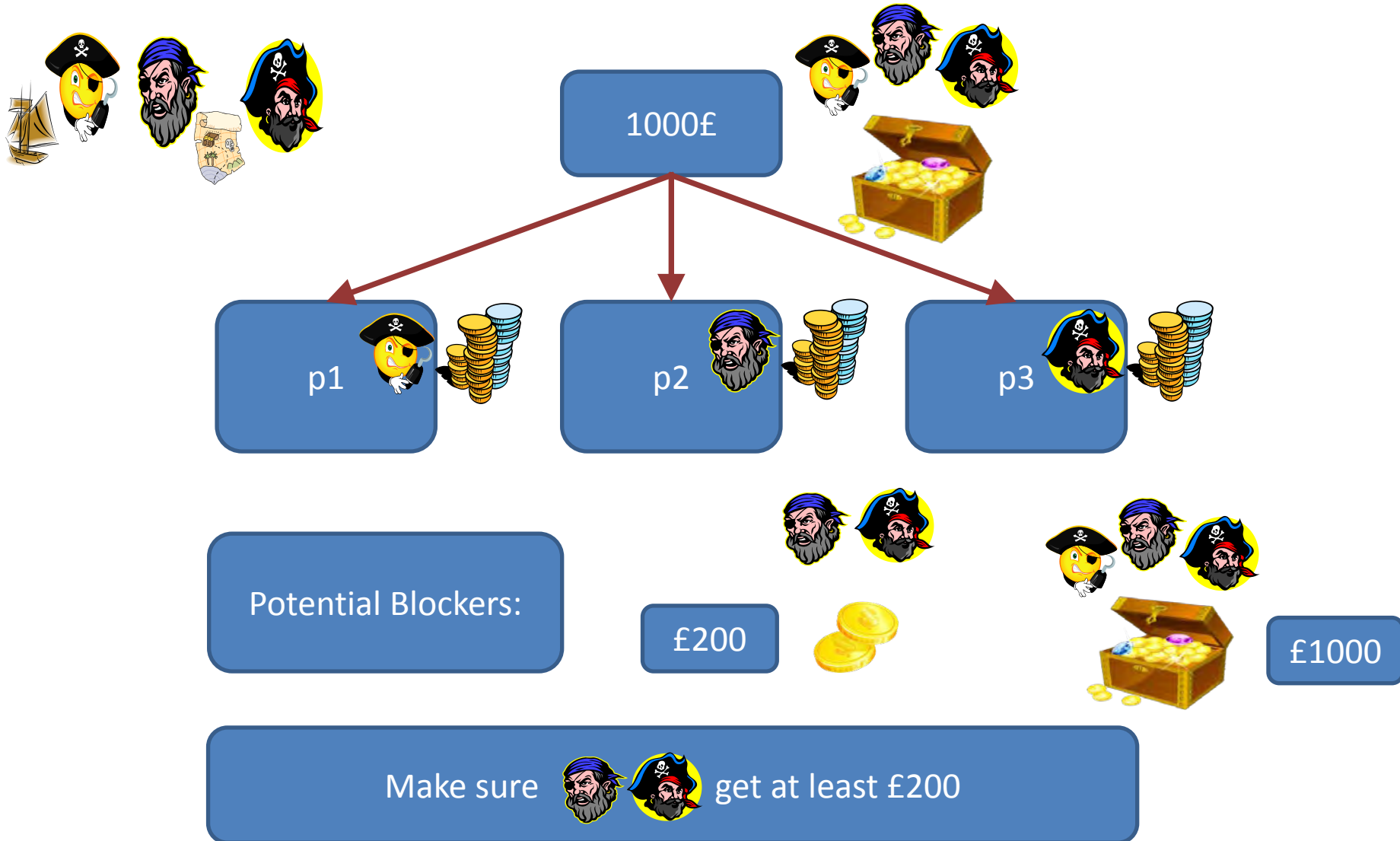


Imputation 	Blocked or Stable?
(900,50,50)	X
(100,500,400)	✓
(100,899,1)	✓
(0,1,999)	✓


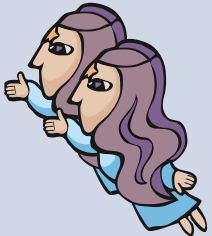
– **Stable** is not always **Fair**!



# Treasure Island – the Core



# Agent properties

	<b>Dummy</b>	Never contributes anything $\forall C, v(C \cup \{a_i\}) = v(C)$
	<b>Equivalent</b>	Contribute equally everywhere $\forall C \neg(a_i \in C) \wedge \neg(a_j \in C) \Rightarrow v(C \cup \{a_i\}) = v(C \cup \{a_j\})$

# Fairness Requirements





- **Dummy axiom:**
  - Dummies get nothing
- **Symmetry axiom:**
  - Equivalent agents get the same
- **Additivity axiom:**
  - Value decomposes over games
- Can we fulfill all of these?



# Marginal Contribution I





Coalition	Value
	£0
	£1000

- Marginal contribution of  to   
– £1000-£0=£1000

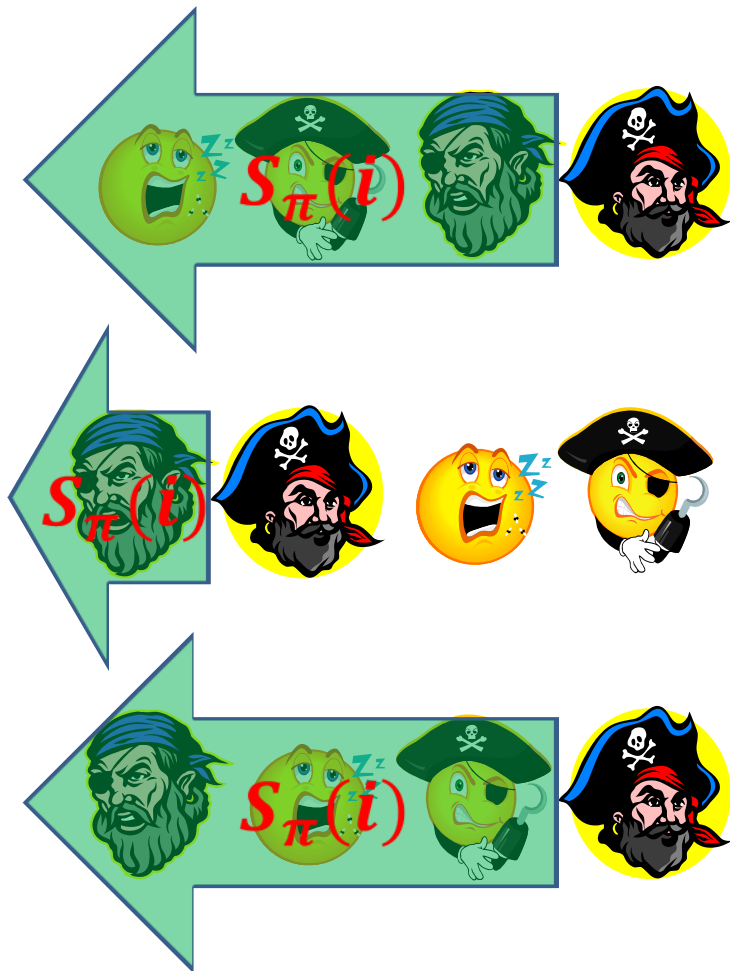
# Marginal Contribution I



Coalition	Value
	£200
	£1000

- Marginal contribution of  to   
—  $£1000 - £200 = £800$

# Permutation Contribution



Before $v(S_\pi(i))$	Including $v(S_\pi(i) \cup \{i\})$	Contribution
£0	£1000	£1000
£0	£200	£200
£0	£1000	£1000

# The Shapley Value: Fairness

- Average contribution across all permutations






















$$sh_i(v) = \frac{1}{n!} \sum_{\pi \in \Pi} [v(s_{\pi}(i) \cup \{i\}) - v(s_{\pi}(i))]$$



- Only fair solution!



# Treasure Island – the Shapley Value

			
  	0	0	1000
  	0	1000	0
  	0	0	1000
  	800	0	200
  	800	200	0
  	0	1000	0
Average	266.66	366.66	366.66



# Power in the UK Elections



Conservatives	Labour	Lib-Dems
306	258	57
66.66%	16.66%	16.66%








Conservatives	Labour	Liberals	Democrats
306	258	28	29
75%	8.33%	8.33%	8.33%

- Game 1: [306, 258, 57; 326]
- Game 2: [306, 258, 28, 29; 326]
- Split makes the Labour less powerful
  - But the power goes to the Conservatives...
  - ... not the Lib-Dems



# The “Rip-off” Game

Playing 2

 Weight: 0.4 Share: <input type="text" value="1.0"/> Team: <input type="text" value="#1"/> ▼	 Weight: 0.4 Share: <input type="text" value="1.0"/> Team: <input type="text" value="#2"/> ▼	 Weight: 0.25 Share: <input type="text" value="1.0"/> Team: <input type="text" value="#3"/> ▼	 Weight: 0.4 Share: <input type="text" value="1.0"/> Team: <input type="text" value="#4"/> ▼	 Weight: 0.25 Share: <input type="text" value="1.0"/> Team: <input type="text" value="#5"/> ▼
--	--	--	--	---

Jackpot: 0.0



# Applications



**COST SHARING**



**POLITICAL POWER**



**AUCTIONS**



**bing™**



# Conclusion



Cooperation

Competition



Thank You

